LITERATURE CITED

1. R. Gardon, J. Am. Ceramic Soc., 39, No. 8, 278-287 (1956).
2. E. N. Bezrukova et al., Thermophysical Properties of Solid Substances [in Russian], Moscow (1973), pp. 97-101.
3. A. A. Men' and Z. S. Settarova, Teplofiz. Vys. Temp., 10, No. 2, 279-284 (1972).
4. V. A. Petrov and S. V. Stepanov, Teplofiz. Vys. Temp., 13, No. 3, 531-537 (1975).
5. V. A. Petrov and S. V. Stepanov, Teplofiz. Vys. Temp., 13, No. 6, 1178-1188 (1975).
6. N. M. Rastorgueva and N. A. Rubtsov, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, Issue 3, No. 13, 10-15 (1981).
7. Z. S. Settarova and O. P. Sergeev, Thermophysical Properties of Solid Substances [in Russian] (1971), pp. 126-128.
8. B. F. Armaly, T. T. Lam, and A. L. Crosbie, AIAA J., 11, No. 11, 1498-1502 (1973).
9. A. L. Crosbie, AIAA J., 17, No. 1, 117-120 (1979).
10. M. N. Otsisik, Complex Heat Transfer [in Russian], Moscow (1976).
11. Yu. V. Lipovtsev and T. A. Betina, Abstracts of Reports, Second Interbranch School on Construction Design Problems [in Russian], Dushanbe (1981), pp. 42-43.
12. Yu. V. Lipovtsev and T. A. Betina, Algorithms for the Solution of Complex Heat Transfer Boundary Value Problems in Emitting and Anisotropically Scattering Media, Computation of the Coefficients of a Two-Flux Scheme [in Russian], Deposited in VINITI, No. 5844-83, Moscow (1983).
13. I. A. Gorban', Teplofiz. Vys. Temp., 18, No. 4, 775-780 (1980).
14. Ivens, Chu, and Churchill, Teploperedacha, No. 5, 69-75 (1965).

NONEQUILIBRIUM COUNTERFLOW CAPILLARY IMPREGNATION

G. I. Barenblatt and A. A. Gil'man

UDC 532.546

A model of capillary impregnation, taking account of disequilibrium of the phase permeabilities, is constructed on the basis of a general scheme of nonequilibrium two-phase filtration proposed in [3]; see also [2].

The theory of counterflow capillary impregnation of a porous medium is constantly under examination by researchers, in particular, in connection with the role played by this process in the displacement of petroleum by water in microinhomogeneous hydrophilic beds. The existing model of capillary impregnation is based on the self-similar solution of [1] (see also [2]), using the classical Muskat–Leverette scheme of the filtration of inhomogeneous liquids. According to this scheme, the relative phase permeabilities of water and petroleum and also the reduced capillary pressure (the Leverette function) are regarded as universal functions of instantaneous saturation σ, which may be determined from data on the steady flow of a mixture of the given composition. However, in reality, the characteristic impregnation time in low-permeability microinhomogeneous blocks may be comparable with the time to establish phase permeabilities and capillary pressure, i.e., the time for regrouping of the liquids along channels of the appropriate dimensions. For this reason, the model of counterflow capillary impregnation must take account of disequilibrium effects.

1. Basic Equations of Model

For the combined filtration of water and petroleum, under broad assumptions, Darcy's law is valid

$$\mathbf{u}_1 = -(k/\mu_1) f_1 \nabla p_1, \quad \mathbf{u}_2 = -(k/\mu_2) f_2 \nabla p_2,$$
$$p_2 - p_1 = T \cos\Theta\, (m/k)^{1/2} J. \tag{1}$$

The simplest formulation of the scheme for taking account of disequilibrium [3, 2] rests on the basis that the functions f_1, f_2, J determined from the data on steady flow of the mixture are monotonic functions of the true water saturation σ. The functions f_1, f_2 vary here

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 3, pp. 456-461, March, 1987. Original article submitted February 14, 1986.

from zero to unity, and J decreases from infinity to zero with increase in σ. Therefore, taking account of disequilibrium in the given scheme reduces to assuming that the functions f_1, f_2, J in a nonequilibrium flow are the same as in the equilibrium flow but are functions not of the true water saturation σ but of some effective water saturation η. (The only assumption here, in fact, is that the effective water saturation η for all three functions f_1, f_2, J is the same.) The kinetic equation between the effective saturation η and the true saturation σ is taken in the form

$$\eta = \sigma + \tau\partial_t\sigma, \tag{2}$$

where τ is a constant for the given rock and liquid vapor, called the substitutional time. The conservation equations for the two components are reduced to the system

$$m\partial_t\sigma + \nabla\{\mathbf{v}F(\eta) + (T\cos\Theta/\mu_2)(mk)^{1/2}F(\eta)f_2(\eta)\nabla J(\eta)\} = 0, \quad \nabla\mathbf{v} = 0. \tag{3}$$

Here the function F is determined by the relation

$$F(\eta) = f_1(\eta)/[f_1(\eta) + (\mu_1/\mu_2)f_2(\eta)], \tag{4}$$

and $\mathbf{v} = \mathbf{u}_1 + \mathbf{u}_2$ is the total flux of liquid. For counterflow capillary impregnation, the total flux is zero, $\mathbf{v} = 0$, and the basic equation for the saturation takes the form

$$\partial_t\sigma = \varkappa\Delta\Phi(\sigma + \tau\partial_t\sigma). \tag{5}$$

Here

$$\varkappa = \frac{T\cos\Theta}{\mu_2}\left(\frac{k}{m}\right)^{1/2}, \quad \Phi(\eta) = -\int_0^\eta F(\eta)f_2(\eta)J'(\eta)\,d\eta. \tag{6}$$

Since F, $f_2 > 0$, $J' < 0$, the function $\Phi(\eta)$ is positive and monotonically increasing. Further, since the function $F(\eta)$ tends rapidly to zero as $\eta \to 0$, like $f_2(\eta)$ as $\eta \to 1$, the function $\Phi(\eta)$ has a high-order zero as $\eta \to 0$ and a maximum when $\eta = 1$.

Within the framework of classical Muskat–Leverette equilibrium theory (see [2], for example), the substitutional time τ is zero and the functions f_1, f_2, F, J are universal functions of the instantaneous water saturation σ. In this case, the Ryzhik equation is valid for the water saturation in capillary impregnation

$$\partial_t\sigma = \varkappa\Delta\Phi(\sigma). \tag{7}$$

The difference between Eqs. (5) and (7) is fundamental: in fact, Eq. (5) is insoluble relative to the time derivative. Multiplying Eq. (5) by τ, differentiating with respect to the time, and adding the result to Eq. (5), a third-order equation is obtained for the effective water saturation η

$$\partial_t\eta = \varkappa\Delta\Phi(\eta) + \varkappa\tau\partial_t\Delta\Phi(\eta). \tag{8}$$

Equation (8) is basic to the proposed model. It generalizes the well-known equation for the pressure p in the filtration of homogeneous liquid in a cracked–porous medium [4] (see also [2])

$$\partial_t p = \varkappa\Delta p + \varkappa\tau\partial_t\Delta p. \tag{9}$$

2. Boundary Problems for the Equations of Nonequilibrium Capillary Impregnation

The basic system of equations for the effective and true water saturation is written in the form

$$\varkappa\tau\Delta\Phi(\eta) - (\eta - \sigma) = 0, \quad \tau\partial_t\sigma = \eta - \sigma. \tag{10}$$

As is evident, this system is degenerate; it does not include the derivative of η with respect to the time and the spatial derivatives of σ; this determines the smoothness properties of the solution. The presence of the Laplacian $\Phi(\eta)$ in Eq. (10) requires continuity of the effective saturation η right to the boundary. Introducing the term $\varepsilon\tau\partial_t\eta$ on the right-hand side of the first relation in Eq. (10), where $\varepsilon \to 0$ is a small parameter, gives

$$\varkappa\tau\Delta\Phi(\eta) - (\eta - \sigma) = \varepsilon\tau\partial_t\eta. \tag{11}$$

The appearance of the time derivative in Eq. (11) entails specifying the effective saturation $\eta(\mathbf{x}, 0)$ at the initial moment. At the onset of impregnation $t = 0$, the true water saturation in the block is also specified

$$\sigma(\mathbf{x}, 0) = \sigma_0(\mathbf{x}). \tag{12}$$

At the boundary of the block, the true water saturation may be regarded as specified as a function of the point of the boundary and the time; water rapidly covers the boundary of the block and the block is immersed in mixture of the given composition. Therefore, in view of Eq. (2), it may be supposed that the effective saturation is also specified at the boundary. In particular, if the block is immersed in pure water from the very beginning, an effective water saturation η of unity is established at the boundary. Equation (11) is a nonlinear parabolic equation in which the "fast" time t/ε appears. Therefore, at a time that is large relative to $\varepsilon\tau$ but small relative to the characteristic time of change in the true saturation τ, the solution $\eta = \eta_0(\mathbf{x})$ satisfying the equation

$$\varkappa\tau\Delta\Phi(\eta_0(\mathbf{x})) - \eta_0(\mathbf{x}) + \sigma_0(\mathbf{x}) = 0 \tag{13}$$

and the boundary condition (at $t = 0$) is established. In particular, if the block is immersed in pure water, the boundary condition takes the form $\eta_0(x) = 1$ at values of x on the boundary of the block Γ.

Knowing boundary and initial conditions, the solution $\eta(x, t)$ giving the distribution of the effective saturation over space at an arbitrary instant of time may be constructed. After determining the effective saturation $\eta(x, t)$, $\sigma(x, t)$ is found from the relation

$$\sigma = \eta - \varkappa\tau\Delta\Phi(\eta). \tag{14}$$

Note that discontinuities in the initial distribution of the true water saturation, in contrast to discontinuities in the effective water saturation (inside the block and at the boundary) do not disappear instantaneously. Since the effective water saturation is continuous, Eq. (2) gives

$$[\sigma] + \tau\partial_t[\sigma] = 0, \quad [\sigma] = [\sigma]_{t=0}\exp(-t/\tau), \tag{15}$$

where the symbol $[\sigma]$ denotes the discontinuity in σ at some point. In particular, if the initial value of the true water saturation at some point of the boundary from inside the block is σ_0, while a value of unity is established from outside the block, the true water saturation at an internal point of the boundary at an arbitrary time t is $1 - (1 - \sigma_0)\exp(-t/\tau)$.

3. Capillary Impregnation of Semiinfinite Block

Consider first a block bounded by the plane $x = 0$ (x is coordinate measured along the normal to the boundary plane) and completely filled with petroleum, in the case when contact with pure water occurs at $t > 0$.

In this case, Eq. (8) takes the form

$$\partial_t\eta = \varkappa\partial^2_{xx}\Phi(\eta) + \varkappa\tau\partial^3_{xxt}\Phi(\eta). \tag{16}$$

It is expedient to switch to the dimensionless independent variables $\xi = x/(\varkappa\tau)^{1/2}$, $\theta = t/\tau$ in this equation; then an equation which does not contain the parameters is obtained

$$\partial_\theta\eta = \partial^2_{\xi\xi}\Phi(\eta) + \partial^3_{\xi\xi\theta}\Phi(\eta). \tag{17}$$

Equation (17) is solved on the semiinfinite interval $0 \leqq \xi < \infty$ when $\theta > 0$. Since $\sigma_0(x) \equiv 0$ in this case, Eq. (13) for the initial distribution of the effective water saturation takes the form

$$d^2\Phi[\eta_0(\xi)]/d\xi^2 - \eta_0(\xi) = 0. \tag{18}$$

The solution of Eq. (18) which is sought is continuous, has a continuous derivative $d\Phi[\eta_0(\xi)]/d\xi$, and satisfies the conditions

$$\eta_0(0) = 1, \quad \eta_0(\infty) = 0. \tag{19}$$

In fact, the above construction of the function $\Phi(\eta)$ is such that the solution $\eta_0(\xi)$ vanishes at the final point ξ_0 and is then identically zero, so that $\eta_0(\xi) \equiv 0$ when $\xi \geqq \xi_0$. (Finiteness of the perturbation region does not apply when $\sigma_0 \neq 0$.) It follows from the condition of continuity of $d\Phi[\eta_0(\xi)]/d\xi$ that this quantity also vanishes when $\xi = \xi_0$. Thus, a

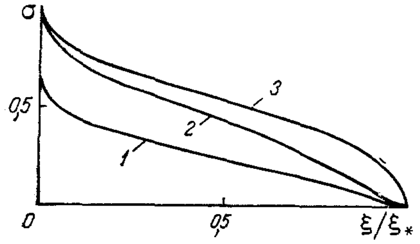

Fig. 1. Evolution of the water-saturation distribution in a semiinfinite sample. Curves 1-3 correspond to dimensionless times $\theta = 1$ (1), 4.7 (2), and ∞ ($\tau = 0$) (3).

nonlinear eigenvalue problem is solved: the second-order Eq. (18) with the three boundary conditions

$$\eta_0(0) = 1, \quad \eta_0(\xi_0) = 0, \quad d\Phi[\eta_0(\xi_0)]/d\xi = 0, \tag{20}$$

and hence the eigenvalue ξ_0 is also found. From the initial condition $\eta(\xi, 0) = \eta_0(\xi)$ obtained and the boundary condition $\eta(0, \theta) = 1$, Eq. (17) is solved. Since the problem has no parameters, the calculation must be performed once for each function $\Phi(\eta)$. The case $\tau = 0$ is degenerate and corresponds to the self-similar solution of [1]: $\sigma = \sigma(\xi\theta^{-1/2})$. The impregnation region at each moment is bounded by the leading front $x_*(t) = (\varkappa\tau)^{1/2}\xi_*(\theta)$; when $\xi \geq \xi_*$, the water saturation is zero. The function $\xi_*(\theta)$ must be determined in the course of solving the problem; $\xi_*(0) = \xi_0$. When $\tau = 0$, $\xi_*(\theta) \sim \theta^{1/2}$. The total amount of water entering the block at each moment is of interest. Per unit cross section of the block, it is

$$\int_0^{x_*} \sigma(x, t)\, dx = (\varkappa\tau)^{1/2} Q(\theta), \quad Q(\theta) = \int_0^{\xi_*} \sigma(\xi, \theta)\, d\xi. \tag{21}$$

Since $\sigma = \eta - \partial_\theta\sigma = \eta - \partial_{\xi\xi}{}^2\Phi(\eta)$ according to Eqs. (2) and (5), the result obtained after integrating and taking account of the continuity of $\partial_\xi\Phi(\eta)$ at $\xi = \xi_*$, i.e., the condition that $\partial_\xi\Phi(\eta) = 0$ when $\xi = \xi_*$, is

$$Q(\theta) = \int_0^{\xi_*} \eta(\xi, \theta)\, d\xi + (\partial_\xi\Phi(\eta))_{\xi=0}. \tag{22}$$

Also, from Eq. (5)

$$Q(\theta) = \int_0^{\xi_*} \sigma(\xi, \theta)\, d\xi = -\int_0^{\theta} (\partial_\xi\Phi(\eta))_{\xi=0} d\theta. \tag{23}$$

Some asymptotic estimates are now obtained. Suppose that at small η the function $\Phi(\eta)$ has a power-law asymptote: $\Phi(\eta) = C\eta^n$. Here C and n are positive constants. Introducing the moving coordinate $\zeta = \xi_*(\theta) - \xi$ measured from the impregnation front, small time intervals in which the velocity of impregnation-front propagation $d\xi_*/d\theta = \lambda$ may be regarded as constant are considered. Suppose that $\eta = D\zeta^p$, $\sigma = K\zeta^q$ close to the front. Substituting this into Eq. (10) in the dimensionless form

$$\partial^2_{\xi\xi}\Phi(\eta) - (\eta - \sigma) = 0, \quad \partial_\theta\sigma = \eta - \sigma, \tag{24}$$

and equating coefficients, it is found that $p = 2/(n - 1)$, $q = (n + 1)/(n - 1)$, $D = ((n - 1)^2/2Cn(n + 1))^{1/(n-1)}$, $K = D/\lambda q$. Hence it follows that the derivative $\partial_\xi\sigma$ vanishes when $\xi = \xi_*$, while the derivative $\partial_\xi\eta$ undergoes a discontinuity from $-\infty$ to zero.

Further, at small θ, the effective water saturation $\eta(\xi, \theta)$ is close to the initial value $\eta_0(\xi)$. Integrating Eq. (18) and using the condition $d\Phi[\eta_0(\xi)]/d\xi = 0$ when $\xi = \xi_*$, it follows that at small θ

$$(\partial_\xi\Phi(\eta))_{\xi=0} \simeq (d\Phi[\eta_0(\xi)]/d\xi)_{\xi=0} = -\int_0^{\xi_0} \eta_0(\xi)\, d\xi, \tag{25}$$

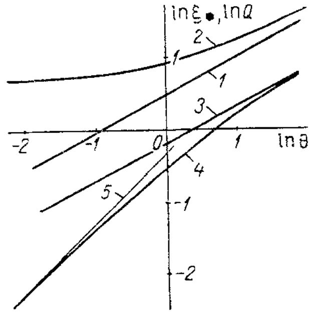

Fig. 2. Dependence of the depth of the impregnation front ξ_* on the time: 1) self-similar case ($\tau = 0$); 2) $\tau \neq 0$; the dependence of the amount of absorbed water $Q(\theta)$ on the time; 3) self-similar case ($\tau = 0$); 4) $\tau \neq 0$; 5) asymptote of $Q(\theta)$ at small times: $Q(\theta) = 0.75\theta$.

and hence from Eq. (23)

$$Q(\theta) = \left(\int_0^{\xi_0} \eta_0(\xi)\, d\xi\right) \theta. \tag{26}$$

As is evident, the asymptote of the water inflow into the block at small times is a linear function of the time.

4. Numerical Calculations

The following model expression for $\Phi(\eta)$ is chosen for the calculations

$$\Phi(\eta) = (15\eta^4 - 24\eta^5 + 10\eta^6)/2, \tag{27}$$

so that $n = 4$, $C = 15/2$, $D = (3/100)^{1/3} = 0.31$, $p = 2/3$, $q = 5/3$. The results of the calculations are shown in Figs. 1 and 2. The distribution of the true water saturation at various dimensionless times θ is shown in Fig. 1, in the universal coordinates σ, ξ/ξ_*. The limiting curve corresponds to the self-similar solution of Eq. (7) in [1]. The time dependence of the impregnation-front coordinate and the amount of water entering the block is shown in Fig. 2, in universal coordinates. The asymptotes at large $\theta = t/\tau \gg 1$ (straight lines 1 and 3) correspond to self-similar solution. Straight line 5 corresponds to the asymptote in Eq. (26) at small times. The curves of $Q(\theta)$ in Fig. 2 allow the influence of the decrease in restructuring time τ on the influx of water into the block to be estimated.

NOTATION

σ, water saturation; η, effective water saturation; u, filtration rate; p, pressure; k, permeability of medium; T, surface tension at water–petroleum–rock triple boundary; Θ, wetting angle; μ, viscosity; f, relative permeability; m, porosity of medium; J, Leverette function; subscript 1 denotes water and subscript 2 petroleum; τ, substitutional time; $F(\sigma)$, saturation function defined by Eq. (4); v, total liquid flux; $\varkappa = T\cos\Theta/(m/k)^{1/2}\mu_2$; σ_0, initial water saturation of sample; ε, small parameter; $\Phi(\eta)$, function defined by Eq. (6); x, spatial coordinate measured along the sample; ξ_*, dimensionless depth of front; Q, dimensionless influx of water in the sample; ξ, θ, dimensionless coordinate and time; C, D, K, p, q, place-determined constants.

LITERATURE CITED

1. V. M. Ryzhik, Izv. Akad. Nauk SSSR, Ser. Mekh. Mashinostr., No. 2, 149-151 (1960).
2. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Motion of Liquids and Gases in Natural Beds [in Russian], Moscow (1984).
3. G. I. Barenblatt and A. P. Vinnichenko, Usp. Mekh., No. 3, 35-50 (1980).
4. G. I. Barenblatt, Yu. P. Zheltov, and T. N. Kochina, Prikl. Mat. Mekh., 24, No. 5, 852-864 (1960).